Practice Test 1

PERTH MODERN SCHOOL

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Semester One 2017
Year 12 Mathematics Methods
Calculator Free

20 Marks   20 Minutes

Name:

1 **(4 marks)**

Show that $\int_1^2 \frac{6x+4}{\sqrt{x}}\,dx = 16\sqrt{2} - 12$.

2 **(3,3,3 marks)**

a) $\frac{dy}{dx} = \frac{2}{x^2} + 4x$, and $y = 3$ when $x = 2$, determine the value of y when $x = 5$

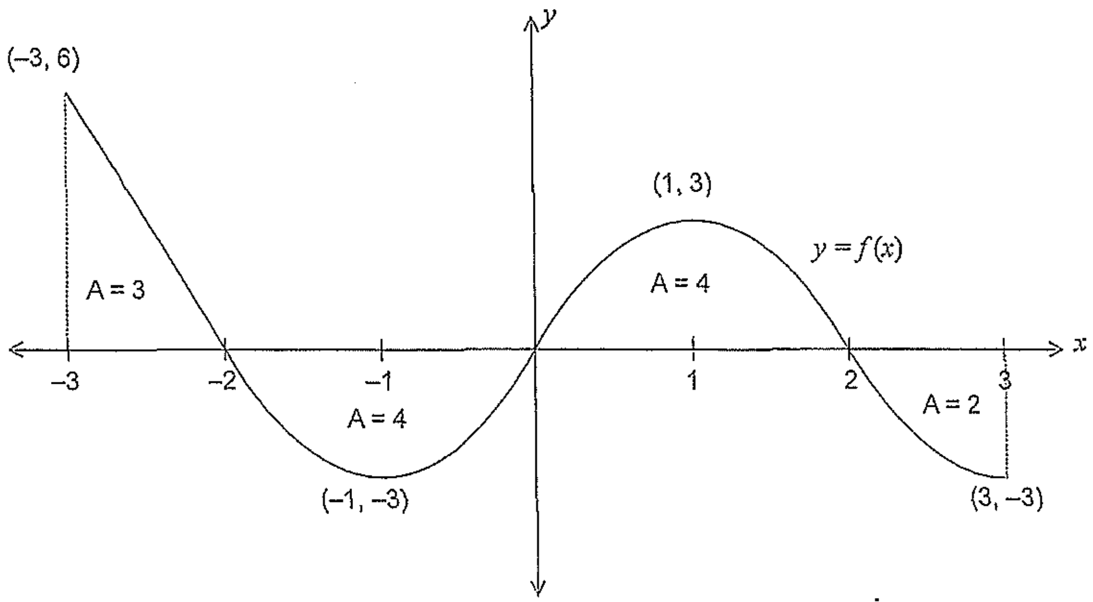
b) *Evaluate* $\int_1^2 \frac{d}{dx}\left(\frac{x^3}{x^2+1}\right) dx$

c) $\frac{d}{dx}\int_4^{x^2} \frac{2}{3t^3-1}\, dt$

3. **[7 marks]** *(3CDMAT 2013:CF7a,b,d)*

The graph of the function $f(x)$ is shown below for $-3 \le x \le 3$.

The areas enclosed between the graph, the x-axis and the lines $x = -3$ and $x = 3$ are marked in the appropriate regions.



Determine:

(a) the value of $\displaystyle\int_{-2}^{3} f(x)\,dx$. [2]

(b) the area enclosed between the graph of $f(x)$ and the x-axis, from $x = -2$ to $x = 3$. [2]

(c) the value of $\displaystyle\int_{0}^{2} (x - f(x))\,dx$. [3]

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Name: Solutions

1 (4 marks)

Show that $\int_1^2 \frac{6x+4}{\sqrt{x}}\,dx = 16\sqrt{2} - 12$.

$= \int_1^2 6x^{\frac{1}{2}} + 4x^{-\frac{1}{2}}\,dx$

$= \left[\frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4x^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^2$

$= \left(4x^{\frac{3}{2}} + 8\sqrt{x}\right)_1^2 = \left(4\cdot(\sqrt{2})^3 + 8\cdot\sqrt{2}\right) - (4\cdot 1 + 8)$

$= 8\sqrt{2} + 8\sqrt{2} - 12$

$= 16\sqrt{2} - 12$

2 (3,3,3 marks)

a) $\frac{dy}{dx} = \frac{2}{x^2} + 4x$, and $y = 3$ when $x = 2$, determine the value of y when $x = 5$

$\frac{dy}{dx} = 2x^{-2} + 4x$

$y = -2x^{-1} + 2x^2 + C$

$= -\frac{2}{x} + 2x^2 + C$

$3 = -\frac{2}{2} + 2\times 4 + C$

$3 = -1 + 8 + C \therefore C = -4$

$y = -\frac{2}{x} + 2x^2 - 4$

$= -\frac{2}{5} + 50 - 4$

$= 45.6$

b) Evaluate $\int_1^2 \frac{d}{dx}\left(\frac{x^3}{x^2+1}\right)dx$

$= \left[\frac{x^3}{x^2+1}\right]_1^2$

$= \frac{2^3}{2^2+1} - \frac{1^3}{1^2+1}$

$= \frac{8}{5} - \frac{1}{2} = \frac{16}{10} - \frac{5}{10} = \frac{11}{10}$

c) $\frac{d}{dx}\int_4^{x^2} \frac{2}{3t^3-1}\,dt$

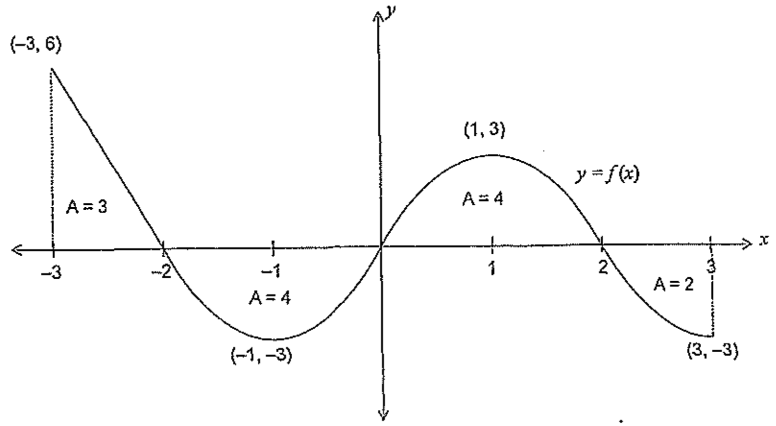
$= \frac{2}{3x^6-1}\cdot 2x$

$= \frac{4x}{3x^6-1}$

3. [7 marks] (3CDMAT 2013:CF7a,b,d)

The graph of the function $f(x)$ is shown below for $-3 \le x \le 3$.

The areas enclosed between the graph, the x-axis and the lines $x = -3$ and $x = 3$ are marked in the appropriate regions.



Determine:

(a) the value of $\int_{-2}^{3} f(x)\,dx$. [2]

$= -4 + 4 + -2 = -2$

(b) the area enclosed between the graph of $f(x)$ and the x-axis, from $x = -2$ to $x = 3$. [2]

$4 + 4 + 2 = 10$

(c) the value of $\int_{0}^{2} (x - f(x))\,dx$. [3]

$\int_0^2 f(x)\,dx = 4$

$= \int_0^2 x\,dx - \int_0^2 f(x)\,dx$

$= \left[\frac{x^2}{2}\right]_0^2 - 4 = 2 - 4 = -2$